Mark Scheme (Results)

October 2021

Pearson Edexcel International A Level
In Further Pure Mathematics F2 (WFM02)
Paper 01

WFM02 Further Pure Mathematics F2 Mark Scheme

Question Number	Scheme	Notes	Marks
1	$z^5 - 32\text{i} = 0 \Rightarrow r^5 = 32 \Rightarrow r = 2$	Correct value for r. May be shown explicitly or used correctly.	B1
	$5\theta = \frac{\pi}{2} + 2n\pi \Rightarrow \theta = \frac{\pi}{10} + \frac{2n\pi}{5}$	Applies a correct strategy for establishing at least 2 values of θ. This can be awarded if if the initial angle $\left(\frac{\pi}{2} \text{ or } \frac{\pi}{10}\right)$ is incorrect but otherwise their strategy is correct.	M1
	$z = 2e^{\text{i}\frac{\pi}{10}},\ 2e^{\text{i}\frac{\pi}{2}},\ 2e^{\text{i}\frac{9\pi}{10}},\ 2e^{\text{i}\frac{13\pi}{10}},\ 2e^{\text{i}\frac{17\pi}{10}}$ or	At least 2 correct, follow through their r	A1ft
	$z = 2e^{\left(\frac{\pi}{10}+\frac{2n\pi}{5}\right)\text{i}}, \quad n = 0,1,2,3,4$	All correct. Must have $r = 2$	A1
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
2	$\dfrac{x}{2-x} \leqslant \dfrac{x+3}{x}$		
Way 1	$\dfrac{x}{2-x} \leqslant \dfrac{x+3}{x} \Rightarrow \dfrac{x}{2-x} - \dfrac{x+3}{x} \leqslant 0$	Collects to one side	M1
	$\dfrac{x}{2-x} - \dfrac{x+3}{x} \leqslant 0 \Rightarrow \dfrac{x^2-(2-x)(x+3)}{x(2-x)} \leqslant 0$ M1: Attempt common denominator A1: Correct fraction		M1 A1
	$x = 0,\ 2$	These critical values	B1
	$x^2-(2-x)(x+3)=0$ $\Rightarrow 2x^2+x-6=0 \Rightarrow x = ...$	Solves the 3TQ in the numerator	M1
	$x = \dfrac{3}{2},\ -2$	These critical values	A1
	$x \leqslant -2,\ 0 < x \leqslant \dfrac{3}{2},\ x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow “or”, “and”, “,” etc. but not $\cap$		A1A1
			(8)
			Total 8
	Alternative 1: $\times x^2(2-x)^2$		
	$x^3(2-x) \leqslant x(x+3)(2-x)^2$	Multiplies by a positive expression	M1
	$x^3(2-x) - x(x+3)(2-x)^2 \leqslant 0$	Collects to one side Correct inequality	M1 A1
	$x = 0,\ 2$	These critical values	B1
	$x(2-x)\left[x^2 - (x+3)(2-x)\right]=0$ $x^2 - (x+3)(2-x)=0$ $\Rightarrow 2x^2+x-6=0 \Rightarrow x = ...$	Attempts to factorise by taking out a factor of $x(2-x)$ and solves resulting 3TQ. May have quartic and apply the factor theorem.	M1
	$x = \dfrac{3}{2},\ -2$	These critical values	A1
	$x \leqslant -2,\ 0 < x \leqslant \dfrac{3}{2},\ x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow “or”, “and”, “,” etc. but not $\cap$		A1A1

Question Number	Scheme	Notes	Marks
3	$w=\frac{(2+\mathrm{i})z+4}{z-\mathrm{i}} \Rightarrow wz-wi=(2+\mathrm{i})z+4$ $\Rightarrow z=...$	Attempts to make z the subject	M1
	$z=\frac{w\mathrm{i}+4}{w-2-\mathrm{i}}$	Correct equation in any form	A1
	$z=\frac{(u+\mathrm{i}v)\mathrm{i}+4}{u+\mathrm{i}v-2-\mathrm{i}}$ $z=\frac{((u+\mathrm{i}v)\mathrm{i}+4)(u-2-(v-1)\mathrm{i})}{(u-2+(v-1)\mathrm{i})(u-2-(v-1)\mathrm{i})}$	Introduces $w = u + \mathrm{i}v$ and multiplies numerator and denominator by the conjugate of the denominator	M1
	$u(v-1)+(4-v)(u-2)=0$	Sets real part = 0 (with or without denominator) Depends on both M marks above	dM1
		Any correct equation	A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 2	$w=\frac{(2+\mathrm{i})z+4}{z-\mathrm{i}}, z=y\mathrm{i} \Rightarrow w=\frac{(2+\mathrm{i})y\mathrm{i}+4}{y\mathrm{i}-\mathrm{i}}$ $w=\frac{(2+\mathrm{i})y\mathrm{i}+4}{y\mathrm{i}-\mathrm{i}}\times\frac{\mathrm{i}}{\mathrm{i}}$	Solves simultaneously and multiplies numerator and denominator by i	M1
	$u=\frac{2y}{y-1}, v=\frac{y-4}{y-1}$	Correct real and imaginary parts	A1
	$u=\frac{2y}{y-1} \Rightarrow y=\frac{u}{u-2}$	Attempts y in terms of u or v	M1
	$y=\frac{u}{u-2} \Rightarrow v=\frac{\frac{u}{u-2}-4}{\frac{u}{u-2}-1}$	Obtains an equation connecting u and v	M1
		Any correct equation	A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			(6)
Way 3	Apply the transformation to any point on the imaginary axis	Eg $(0,0)\to(0,4)$ $(0,1)\to(4,-2)$	M1
	Apply the transformation to a second point on the imaginary axis	This is the second M mark on e-PEN	M1
	Both transformations correct	This is the first A mark on e-PEN	A1
	Complete method to obtain an equation for the line thro' their 2 points in the w-plane		M1
	Correct equation in any form		A1
	$3u+2v-8=0$	Correct equation in the required form (allow any integer multiple)	A1
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$(x+1)\frac{dy}{dx}-xy=e^{3x} \qquad x>-1$		
	$\frac{dy}{dx}-\frac{xy}{(x+1)}=\frac{e^{3x}}{(x+1)}$	Correctly rearranged equation	B1
	$I=e^{\int\frac{-x}{x+1}dx}=e^{\int\left(-1+\frac{1}{x+1}\right)dx}$	Correct strategy for the integrating factor including an attempt at the integration	M1
	$=e^{-x+\ln(x+1)}$	For $-x+\ln(x+1)$	A1
	$=(x+1)e^{-x}$	Correct integrating factor	A1
	$y(x+1)e^{-x}=\int\frac{e^{3x}}{x+1}\times(x+1)e^{-x}\,dx$	Uses their integrating factor to reach the form $yI=\int QI\,dx$	M1
	$y(x+1)e^{-x}=\frac{1}{2}e^{2x}+c$	Correct equation (with or without $+c$)	A1
	$y=\frac{e^{3x}}{2(x+1)}+\frac{ce^{x}}{(x+1)}$	Correct answer (allow equivalent forms). Must have $y=$...	A1
			(7)
(b)	$x=0, y=5\Rightarrow 5=\frac{1}{2}+c\Rightarrow c=\frac{9}{2}$	Substitutes $x=0$ and $y=5$ and attempts to find a value for c.	M1
	$y=\frac{e^{3x}}{2(x+1)}+\frac{9e^{x}}{2(x+1)}$	Cao (oe) Must have $y=$...	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$y=\tan^2 x \Rightarrow \frac{dy}{dx}=2\tan x\sec^2 x$	Correct first derivative any correct form	B1
	$\frac{dy}{dx}=2\tan x\sec^2 x \Rightarrow \frac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x$ M1: Correct application of the product rule and chain rule A1: Correct expression		M1A1
	$\frac{d^2y}{dx^2}=2\sec^4 x+4\sec^2 x\tan^2 x \Rightarrow \frac{d^3y}{dx^3}=8\sec^4 x\tan x+8\sec^2 x\tan^3 x+8\sec^4 x\tan x$ Or $\frac{d^2y}{dx^2}=6\sec^4 x-4\sec^2 x \Rightarrow \frac{d^3y}{dx^3}=24\sec^4 x\tan x-8\sec^2 x\tan x$ M1: Attempt to differentiate using product and chain rule. At least one term to be correct		M1
	$=8\sec^4 x\tan x+8\sec^2 x\tan x\left(\sec^2 x-1\right)+8\sec^4 x\tan x$ $=24\sec^4 x\tan x-8\sec^2 x\tan x=8\sec^2 x\tan x\left(3\sec^2 x-1\right)$ Fully correct expression		A1
			(5)
(b)	$(y)_{\frac{\pi}{3}}=3,\ (y')_{\frac{\pi}{3}}=8\sqrt{3},\ (y'')_{\frac{\pi}{3}}=80,\ (y''')_{\frac{\pi}{3}}=352\sqrt{3}$	Attempts the values up to the third derivative when $x=\frac{\pi}{3}$	M1
	$y=3+8\sqrt{3}\left(x-\frac{\pi}{3}\right)+\frac{80}{2!}\left(x-\frac{\pi}{3}\right)^2+\frac{352\sqrt{3}}{3!}\left(x-\frac{\pi}{3}\right)^3+\ldots$ Correct application of the Taylor series 2! or 2, 3! or 6		M1
	$y=3+8\sqrt{3}\left(x-\frac{\pi}{3}\right)+40\left(x-\frac{\pi}{3}\right)^2+\frac{176\sqrt{3}}{3}\left(x-\frac{\pi}{3}\right)^3+\ldots$ Correct expansion Must start y = ... or $\tan^2 x$ = ... f(x) only accepted if f(x) has been defined to be $\tan^2 x$		A1
			(3)
			Total 8

Question Number	Scheme	Notes	Marks
6(a)	$\lvert z+1-13\mathrm{i}\rvert = 3\lvert z-7-5\mathrm{i}\rvert \Rightarrow (x+1)^2+(y-13)^2 = 9\{(x-7)^2+(y-5)^2\}$ Correct application of Pythagoras Accept 3 or 9 on RHS		M1
	$\Rightarrow x^2+y^2-16x-8y+62=0$	Correct equation in any form with terms collected	A1
	Centre (8, 4)	Correct centre. i included scores A0	A1
	$r^2 = 64+16-62 = ...$	Correct method for r or r^2	M1
	$r=\sqrt{18}$ or $3\sqrt{2}$	Correct radius. Must be exact.	A1
			(5)
(b)	$\arg(z-8-6\mathrm{i}) = -\dfrac{3\pi}{4} \Rightarrow y-6 = x-8$	Converts the given locus to the correct Cartesian form	B1
	$\Rightarrow x^2+y^2-16x-8y+62=0$ $\Rightarrow x^2+(x-2)^2-16x-8(x-2)+62=0 \Rightarrow x=...$ or $\Rightarrow (y+2)^2+y^2-16x-8(y+2)+62=0 \Rightarrow y=...$	Uses both Cartesian equations to obtain an equation in one variable and attempts to solve	M1
	$x=7-2\sqrt{2}$ or $y=5-2\sqrt{2}$	One correct "coordinate"	A1
	R is $7-2\sqrt{2}+(5-2\sqrt{2})\mathrm{i}$ **or** $x=7-2\sqrt{2}$ and $y=5-2\sqrt{2}$	Correct complex number or coordinates and no others. Must be exact	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
7(a)	$x = t^2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = 2t\frac{\mathrm{d}t}{\mathrm{d}y}$ oe	Correct application of the chain rule	M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2t}\frac{\mathrm{d}y}{\mathrm{d}t}\left(\text{or e.g. } \frac{1}{2\sqrt{x}}\frac{\mathrm{d}y}{\mathrm{d}t}\right)$	Any correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or equivalent equation	A1
	$2\sqrt{x}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \Rightarrow x^{-\frac{1}{2}}\frac{\mathrm{d}y}{\mathrm{d}x} + 2\sqrt{x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ (NB $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 4x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$)	Fully correct strategy to obtain an equation involving $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ Chain rule used on at least one term. Depends on the first M mark	dM1
	$4x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\left(1+2\sqrt{x}\right)\frac{\mathrm{d}y}{\mathrm{d}x} - 15y = 15x \Rightarrow 4x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\sqrt{x}\frac{\mathrm{d}y}{\mathrm{d}x} + 2\frac{\mathrm{d}y}{\mathrm{d}x} - 15y = 15x$ $\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} - 15y = 15t^2$ * ddM1: Substitutes into the given differential equation. The full substitution must be seen. Depends on both M marks. A1*: Cso		ddM1 A1*
			(5)
(b)	$m^2 + 2m - 15 = 0 \Rightarrow m = 3, -5$	Attempts to solve $m^2 + 2m - 15 = 0$	M1
	$y = A\mathrm{e}^{-5t} + B\mathrm{e}^{3t}$	Correct CF	A1
	$y = at^2 + bt + c \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 2at + b \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 2a$ $\Rightarrow 2a + 4at + 2b - 15at^2 - 15bt - 15c = 15t^2$ Starts with the correct PI form and differentiates twice and substitutes		M1
	$-15a = 15 \Rightarrow a = \ldots$ $4a - 15b = 0 \Rightarrow b = \ldots$ $2a + 2b - 15c = 0 \Rightarrow c = \ldots$	Complete method to find *a*, *b* and *c* by comparing coefficients. Values for all 3 needed. Depends on the second M mark.	dM1
	$y = A\mathrm{e}^{-5t} + B\mathrm{e}^{3t} - t^2 - \frac{4}{15}t - \frac{38}{225}$	Correct GS. Must start $y = \ldots$	A1
			(5)
(c)	$y = A\mathrm{e}^{-5\sqrt{x}} + B\mathrm{e}^{3\sqrt{x}} - x - \frac{4}{15}\sqrt{x} - \frac{38}{225}$	Correct equation (follow through their answer to (b)) Must start $y = \ldots$	B1ft
			(1)
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$x = r\cos\theta = (1+\sin\theta)\cos\theta$ $\Rightarrow \frac{dx}{d\theta} = \cos^2\theta - (1+\sin\theta)\sin\theta$ or $\Rightarrow \frac{dx}{d\theta} = -\sin\theta + \cos 2\theta$	Differentiates $r\cos\theta$ using product rule or double angle formula	M1
		Correct derivative in any form	A1
	$\cos^2\theta - (1+\sin\theta)\sin\theta = 0 \Rightarrow 1-\sin^2\theta - \sin\theta - \sin^2\theta = 0 \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ or $-\sin\theta + \cos 2\theta = 0 \Rightarrow -\sin\theta + 1 - 2\sin^2\theta = 0 \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ Sets $\frac{dx}{d\theta} = 0$ and proceeds to a 3TQ in $\sin\theta$ Depends on the first M mark		dM1
	$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$ $\Rightarrow \sin\theta = \frac{1}{2}, (-1) \Rightarrow \theta = ...$	Solves for θ. Depends o both M marks above.	ddM1
	$\left(\frac{3}{2}, \frac{\pi}{6}\right)$	Correct coordinates and no others. Need not be in coordinate brackets.	A1
			(5)
(b)	$\int (1+\sin\theta)^2\, d\theta = \int (1 + 2\sin\theta + \sin^2\theta)\, d\theta$ $= \int \left(1 + 2\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$	Attempts $\left(\frac{1}{2}\right)\int r^2\, d\theta$ and applies $\sin^2\theta = \pm\frac{1}{2} \pm \frac{1}{2}\cos 2\theta$ Ignore any limits shown	M1
	$\int (1+\sin\theta)^2\, d\theta = \frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$	Correct integration (Ignore limits)	A1
	$\frac{1}{2}\left[\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{1}{2}\left[\frac{3\pi}{4} - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8}\right)\right] \left(= \frac{\pi}{4} + \frac{9\sqrt{3}}{16}\right)$	Applies the limits of $\frac{\pi}{2}$ and their $\frac{\pi}{6}$ Substitution must be shown but no simplification needed	M1
	Trapezium: $\frac{1}{2}\left(2 + \left(2 - \frac{3}{2}\sin\frac{\pi}{6}\right)\right) \times \frac{3}{2}\cos\frac{\pi}{6}$ $\left(= \frac{39\sqrt{3}}{32}\right)$	Uses a correct strategy for the area of trapezium *OQSP*	M1
	Area of $R = \frac{39\sqrt{3}}{32} - \frac{\pi}{4} - \frac{9\sqrt{3}}{16}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32}\left(21\sqrt{3} - 8\pi\right)$	Cao	A1
			(6)

		Total 11

Question Number	Scheme	Notes	Marks
9(a)	$n^5-(n-1)^5=n^5-\left(n^5-5n^4+10n^3-10n^2+5n-1\right)=...$ Starts the proof by expanding the bracket		M1
	$5n^4-10n^3+10n^2-5n+1$*	Correct proof with no errors. Full expansion of $(n-1)^5$ must be shown.	A1*
			(2)
(b)	$1^5-0^5=5(1)^4-10(1)^3+10(1)^2-5(1)+1$ $2^5-1^5=5(2)^4-10(2)^3+10(2)^2-5(2)+1$ .... $(n-1)^5-(n-2)^5=5(n-1)^4-10(n-1)^3+10(n-1)^2-5(n-1)+1$ $(n)^5-(n-1)^5=5(n)^4-10(n)^3+10(n)^2-5(n)+1$ $n^5=5\sum_{r=1}^{n}r^4-10\sum_{r=1}^{n}r^3+10\sum_{r=1}^{n}r^2-5\sum_{r=1}^{n}r+n$ M1: Applies the result from part (a) between 1 and *n* and sums **both** sides Min 3 lines shown A1: Correct equation If only the last line is seen, award M1A1 These marks can be implied by a correct following stage.		M1A1
	$n^5=5\sum_{r=1}^{n}r^4-10\times\frac{1}{4}n^2(n+1)^2+10\times\frac{1}{6}n(n+1)(2n+1)-5\times\frac{1}{2}n(n+1)+n$ M1: Introduces at least 2 correct summation formulae A1: Correct equation		M1A1
	$5\sum_{r=1}^{n}r^4=\frac{5}{2}n^2(n+1)^2-\frac{5}{3}n(n+1)(2n+1)+\frac{5}{2}n(n+1)+n^5-n=...$ $5\sum_{r=1}^{n}r^4=n(n+1)\left[\frac{5}{2}n(n+1)-\frac{5}{3}(2n+1)+\frac{5}{2}+n^3-n^2+n-1\right]$ Makes $5\sum_{r=1}^{n}r^4$ or $\sum_{r=1}^{n}r^4$ the subject and takes out a factor of $n(n+1)$		M1
	$\sum_{r=1}^{n}r^4=\frac{1}{30}n(n+1)\left[15n(n+1)-10(2n+1)+15+6\left(n^3-n^2+n-1\right)\right]$ $=\frac{1}{30}n(n+1)\left[6n^3+9n^2+n-1\right]=\frac{1}{30}n(n+1)(2n+1)(...)$ Takes out a factor of $n(n+1)(2n+1)$ Depends on all previous method marks		dM1
	$=\frac{1}{30}n(n+1)(2n+1)\left(3n^2+3n-1\right)$	cao	A1
			(7)
			Total 9